

Mechanical Model for Relativistic Blast Waves and Stratified Fireballs

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Abstract. We propose a simple mechanical model for relativistic explosions with both forward and reverse shocks, which allows one to do fast calculations of GRB afterglow. The blast wave in the model is governed by pressures P_F and P_R at the forward and reverse shocks. We show that the simplest assumption $P_F = P_R$ is in general inconsistent with energy conservation law. The model is applied to GRBs with non-uniform ejecta. Such "stratified fireballs" are likely to emerge with a monotonic velocity profile after an internal-shock stage. We calculate the early afterglow emission expected from stratified fireballs.

INTRODUCTION

Relativistic blast waves from GRBs are believed to produce the observed afterglow emission of the bursts. The blast-wave structure is schematically shown in Figure 1. To a reasonably good approximation, the whole blast [the region between FS and RS, including the contact discontinuity (CD)] may be assumed to have a common Lorentz factor γ_s [1]. The Lorentz factors of the forward shock (FS) and reverse shock (RS) are denoted by γ_F and γ_R , respectively. Pressures P_F and P_R behind the forward and reverse shocks are calculated from the jump conditions.

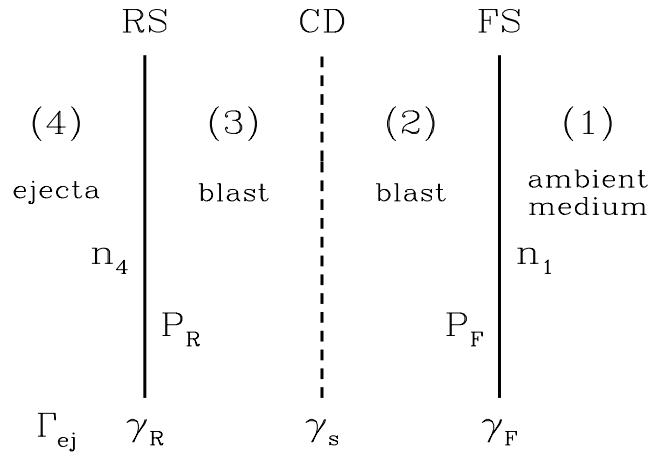


FIGURE 1. An illustrative diagram of 4 regions in a blast wave. Here n_1 and n_4 are the number densities of the ambient medium and the ejecta, respectively.

STRATIFIED FIREBALL

We focus here on the early stages of the GRB explosion when both forward and reverse shocks are expected to produce significant emission. Our calculations assume spherical symmetry. However, the results also apply to beamed explosions at early stages, when $\gamma_s > \theta_{jet}^{-1}$ where θ_{jet} is the opening angle of the ejecta.

The reverse shock (RS) is expected to propagate in the ejecta and produce significant emission at radii $r \sim 10^{16} - 10^{17}$ cm. The RS emission depends on the structure of the ejecta that forms at smaller radii $r < 10^{16}$ cm, when internal shocks take place [1].

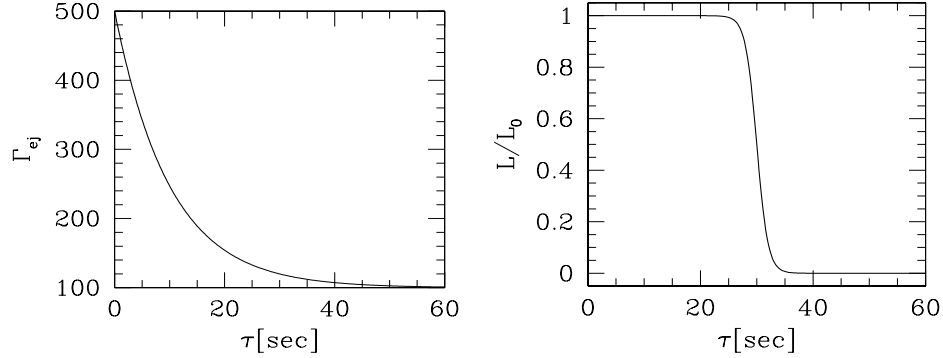


FIGURE 2. Lorentz factor (**Left**) and kinetic luminosity (**Right**) of ejecta

Theoretically, one expects the ejecta to emerge with a monotonic velocity profile (internal shocks tend to make the profile monotonic). Therefore we study here radially stratified ejecta and calculate the emission expected from such “stratified fireballs.”

A simple example model considered here assumes ejecta with radial profile of Lorentz factor $\Gamma_{ej}(\tau) = (\gamma_1 - \gamma_2)e^{-a\tau/\tau_b} + \gamma_2$ shown in Figure 2 ($\gamma_1 = 500$, $\gamma_2 = 100$, $a = 3$, and $\tau_b = 30$ s is the burst duration). Here τ is the Lagrangian coordinate that labels the shells in the ejecta by their ejection times. The luminosity of the ejecta $L(\tau) = 4\pi r^2 v_{ej} \Gamma_{ej}^2 n_4 m_p c^2$ is also shown in Figure 2. Specifically, $L(\tau) = L_0 / (1 + \exp[(\tau - \tau_b)/\tau_d])$ is assumed here with $L_0 = 10^{52}$ erg/s and $\tau_d = 1$ s for the timescale of decay. The total energy of the burst is given by $E_{ej} = L_0 \tau_d \ln(1 + e^{\tau_b/\tau_d})$.

MODELING THE BLAST WAVE

1. Customary approximation: $P_F = P_R$

The customary approximate model blast wave assumes equal pressures at the forward and reverse shocks: $P_F = P_R$ [1]. This assumption, as we show in an accompanying paper, enables a simple analytical solution for γ_s and pressure $P = P_F = P_R$:

$$\gamma_s = \Gamma_{ej} \left[(1 - 2\sigma) + 2(\sigma^2 + (\Gamma_{ej}^2 - 1)\sigma)^{1/2} \right]^{-1/2} \quad \text{where} \quad \sigma = \frac{n_1}{n_4},$$

$$P = \frac{4}{3}(\gamma_s^2 - 1)n_1 m_p c^2,$$

It turns out however that the assumption of equal pressure leads to a contradiction with the law of energy conservation. The total energy of the blast wave and ejecta at a time t may be found by integrating the 00 component of the stress-energy tensor over volume, $E = \int T_{00} dV$. The result is shown in Figure 3. E decreases with time (dashed curve in the left panel), i.e. the energy is not conserved. We therefore propose another dynamical description.

2. Consistent mechanical model

Instead of finding γ_s from $P_F = P_R$ we solve the differential equation for γ_s :

$$\frac{d\gamma_s}{dt} = \frac{1}{\gamma_s H} v_s (P_R - P_F) - \frac{1}{H} (\gamma_s - 1/\gamma_s) \int_{RS}^{FS} \left(\frac{dP}{dt} \right) dr,$$

where $H = \int (U + P) dr$ is the integrated enthalpy of the blast. The first term on the right describes the external force applied to the blast due to the difference between P_F and P_R , and the second term describes the adiabatic acceleration of the blast. This equation is derived from the exact relativistic fluid equation. It is obtained by integrating over the blast and may be viewed as a mechanical equation. The mechanical model is consistent with the energy and momentum conservation (solid curve in the left panel of Fig. 3).

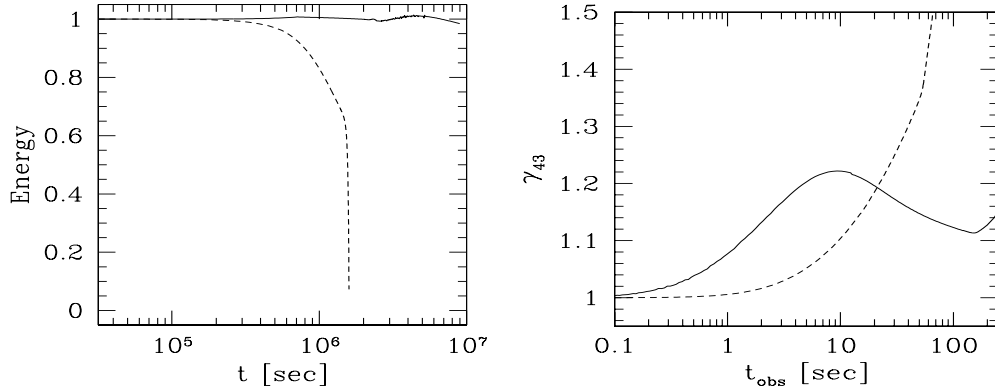


FIGURE 3. **Left:** Dashed curve shows the total energy of the blast wave calculated using $P_F = P_R$ assumption, solid curve – using our mechanical model. **Right:** Lorentz factor γ_{43} of region 4 relative to region 3 for the stratified (solid curve) and non-stratified (dashed curve) fireballs; our mechanical model is used in the calculations.

3. Light curves from stratified fireballs

We calculate the afterglow emission using the standard simple model of the postshock plasma. It assumes that a fraction $\epsilon_e \sim 0.1$ of the shock energy goes to the electrons and accelerates them with a power-law energy spectrum $dN/dE_e \propto E_e^{-P}$. The postshock magnetic field is parameterized by the “equipartition” parameter $\epsilon_B = (B^2/8\pi)U_{th}^{-1} < 1$ where U_{th} is the thermal energy density of the shocked medium.

We discretize the external medium and the ejecta into spherical mass shells m_i and use the Lagrangian description of the blast wave. Each m_i is impulsively heated at some point by a shock front (forward or reverse). We track the subsequent evolution of the shell and calculate its synchrotron emission in the shell comoving frame.

The shock-heated shell suffers radiative and adiabatic energy losses in the expanding blast wave, and its magnetic field decreases, so the shell emission weakens with time. We find its contribution to observed radiation at a given observed time t_{obs} . Integration over all m_i gives the exact total spectrum received from the blast wave at a given t_{obs} .

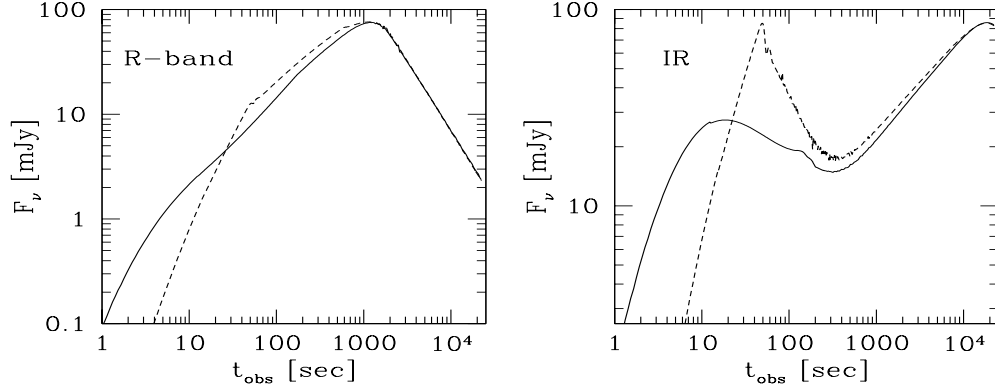


FIGURE 4. **Left:** Optical light curve (R-band) from stratified (solid) and non-stratified (dashed) fireballs. The explosion parameters are $\epsilon_B = 10^{-3}$, $\epsilon_e = 0.1$, $p = 2.5$, $n_1 = 10 \text{ cm}^{-3}$, and redshift $z = 1$. **Right:** IR light curve ($\nu_{obs} = 10^{13} \text{ Hz}$) from the same models.

The optical and infrared light curves for two example models (stratified and non-stratified) are shown in Figure 4. We find weak optical emission from the reverse shock. It is weak because the RS Lorentz factor relative to the blast, γ_{43} , is modest and the emitted spectrum peaks in IR rather than in optical band. The peak frequency is

$$\nu_m \approx 10^6 B \gamma_m^2 \text{ Hz}, \quad \gamma_m = 1 + \frac{p-2}{p-1} \frac{m_p}{m_e} \epsilon_e (\gamma_{43} - 1).$$

The evolution of γ_{43} is shown in Figure 3 (right panel); its typical value is 1.2 and the factor $(\gamma_{43} - 1)^2$ that enters ν_m is small, $\sim 1/20$, shifting the peak frequency to IR.

The behavior of the RS Lorentz factor γ_{43} in the stratified fireball is significantly different from the non-stratified case: it reached maximum and then minimum (Fig. 3). Therefore, the RS emission in the stratified fireball is weaker and its peak in the light curve is less pronounced, with a smoother rise and slower decay.

ACKNOWLEDGMENTS

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